Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

The effect of the biaxial permittivity tensor and tilted layer geometries on the switching of ferroelectric liquid crystals

M. J. Towler[a]; J. C. Jones[a]; E. P. Raynes[a]
[a] Electronics Division of the Defence Research Agency, Worcs, England

To cite this Article Towler, M. J. , Jones, J. C. and Raynes, E. P.(1992) 'The effect of the biaxial permittivity tensor and tilted layer geometries on the switching of ferroelectric liquid crystals', Liquid Crystals, 11: 3, 365 — 371
To link to this Article: DOI: 10.1080/02678299208028995
URL: http://dx.doi.org/10.1080/02678299208028995

LIQUID CRYSTALS, 1992, VOL. 11, NO. 3, 365–371

The effect of the biaxial permittivity tensor and tilted layer geometries on the switching of ferroelectric liquid crystals

by M. J. TOWLER, J. C. JONES and E. P. RAYNES*
Electronics Division of the Defence Research Agency, RSRE,
St Andrews Road, Gt Malvern, Worcs WR14 3PS, England

(Received 3 June 1991; accepted 16 October 1991)

Using a simple uniform switching model, we investigate the behaviour of the voltage dependent switching time of surface stabilized ferroelectric liquid crystals as a function of the biaxial permittivity tensor and the layer tilt angle. We show that the dielectric biaxiality can markedly effect the response time (τ) of the device and is the origin of the minimum in τ as a function of voltage (V) in tilted layer systems. The dielectric biaxiality should, therefore, be optimized for multiplexing schemes which use the τ–V minimum.

1. Introduction

Surface stabilized ferroelectric liquid crystals [1, 2] reorientate on the application of a DC electric field due to the coupling between the spontaneous polarization and the applied electric field. The surface stabilized ferroelectric liquid crystal has enchevroned layers [3] and a non-uniform director profile [4], however under an applied electric field it is reasonable to assume a uniform switching model so the reorientation process is described by a simple rotation around the S_C cone, thus

$$\eta \frac{d\phi}{dt} = P_s \frac{V}{d} \cos\delta \sin\phi, \tag{1}$$

where η is the rotational viscosity, P_s is the spontaneous polarization, V is the applied voltage, d is the cell thickness, δ is the layer tilt angle and ϕ is the azimuthal angle; $\phi = 0$, π are the two fully switched states. Equation (1) indicates that higher P_s materials will switch more quickly, and are therefore desirable. However in complex multiplexed displays a high spontaneous polarization can lead to a reduction in bistability, due to either ionic impurities [5] or the polarization surface charge [6]. High P_s materials are also more likely to show half splayed states [7], which do not in general show extinction between crossed polarizers. The presence of half splayed states may or may not be removed by the data voltage and can, therefore, degrade display appearance.

Low P_s materials show a minimum in response time (τ) as a function of the applied voltage [8], due to electric field coupling to the dielectric anisotropy dominating the coupling to P_s at higher voltages. Multiplexing schemes showing high contrast ratios ($CR > 40:1$) have been designed to utilize this anomalous switching behaviour [9, 10] and so ideally the minimum in the τ–V curve should occur at low voltages and short times. It is, therefore, desirable to understand the origin and position of the minimum in the τ–V curve in detail.

* Author for correspondence.

2. Theory

Defining the principal axes of the biaxial permittivity tensor [11] as, ε_3 along the director, ε_2 along the C_2 axis of symmetry and ε_1 normal to these, for a parallel plate capacitor we have [12]

$$\varepsilon=\varepsilon_1(\cos\theta\cos\delta\sin\phi+\sin\theta\sin\delta)^2+\varepsilon_2\cos^2\delta\cos^2\phi$$
$$+\varepsilon_3(\sin\theta\cos\delta\sin\phi-\cos\theta\sin\delta)^2, \qquad (2)$$

where θ is the permittivity defined cone angle. Although the two fold rotation axis is the only well-defined principal axis, it is reasonable to assume that θ is equivalent to the cone angle measured optically. Defining the dielectric anisotropies as $\Delta\varepsilon=\varepsilon_3-\varepsilon_1$ and $\partial\varepsilon=\varepsilon_2-\varepsilon_1$ we obtain

$$\varepsilon=\varepsilon_1+\Delta\varepsilon(\sin\theta\cos\delta\sin\phi-\cos\theta\sin\delta)^2+\partial\varepsilon\cos^2\delta\cos^2\phi. \qquad (3)$$

The reorientation is then described by (see Appendix)

$$\eta\frac{d\phi}{dt}=P_s\frac{V}{d}\cos\delta\sin\phi+\varepsilon_0\frac{V^2}{d^2}$$
$$\times\left[(\Delta\varepsilon\sin^2\theta-\partial\varepsilon)\cos^2\delta\sin\phi\cos\phi-\frac{\Delta\varepsilon}{4}\sin2\theta\sin2\delta\cos\phi\right]. \qquad (4)$$

2.1. *Simple cases*

We consider first some special cases of equation (4) to demonstrate the importance of the dielectric anisotropies to the occurrence of the τ–V minimum.

2.1.1. *$\delta=0$ The bookshelf geometry*

Equation (4) reduces to

$$\eta\frac{d\phi}{dt}=P_s\frac{V}{d}\sin\phi+\varepsilon_0\frac{V^2}{d^2}(\Delta\varepsilon\sin^2\theta-\partial\varepsilon)\sin\phi\cos\phi. \qquad (5)$$

By direct comparison to earlier work [13], a minimum time (τ_{min}) for complete switching occurs for the applied voltage

$$|V_{\min}|=\left|\frac{P_sd}{\varepsilon_0\sqrt{3(\Delta\varepsilon\sin^2\theta-\partial\varepsilon)}}\right|, \qquad (6a)$$

with

$$\tau_{\min}\propto\frac{\eta(\Delta\varepsilon\sin^2\theta-\partial\varepsilon)}{P_s^2}. \qquad (6b)$$

2.1.2. *$\Delta\varepsilon=0$ $\partial\varepsilon\neq0$*

We have

$$\eta\frac{d\phi}{dt}=P_s\frac{V}{d}\cos\delta\sin\phi-\varepsilon_0\frac{V^2}{d^2}\partial\varepsilon\cos^2\delta\sin\phi\cos\phi; \qquad (7)$$

thus a minimum time for complete reorientation occurs for

$$|V_{\min}|=\left|\frac{P_sd}{\sqrt{3}\varepsilon_0\,\partial\varepsilon\cos\delta}\right|, \qquad (8a)$$

with

$$\tau_{\min} \propto \frac{\eta \partial\varepsilon}{P_s^2}. \tag{8 b}$$

The behaviour described by equations (5) and (7) is simple and is easy to understand. At low voltages the torque is proportional to $\sin\phi$ and the director can switch between $\phi=0$ and $\phi=\pi$; but as the applied voltage is increased the torque term proportional to $-\sin\phi\cos\phi$ (given that $\Delta\varepsilon<0$ and $\partial\varepsilon>0$) starts to dominate slowing the response of the system dramatically [13, 14].

Both of these cases highlight some important features, but for simplicity we consider equations (8 *a*) and (8 *b*). In practice a device is usually required to operate at a given voltage and so the P_s of a particular ferroelectric liquid crystal mixture is adjusted to yield the correct $V_{\min}$, from equation (8 *a*) this implies the required $P_s \propto \partial\varepsilon$ and from equation (8 *b*) this leads to $\tau_{\min} \propto 1/\partial\varepsilon$, hence $\partial\varepsilon$ is preferred to be large.

2.2. *The general case*

In practice we are interested in the general case described by equation (4). Experimentally a clean switching criterion is often used as a measure of the response time for a given voltage [13]. In a simple uniform model this corresponds to the time to switch from an initial position ϕ_0 to the position $\phi=\pi/2$. For low surface pretilt, ϕ_0 can be any angle between the in-plane condition ϕ_p and the infinite voltage AC field stabilized position ϕ_{ac} [12], with

$$\sin\phi_{ac} = \frac{\sin\theta\cos\theta\tan\delta}{b^2+\sin^2\theta}, \tag{9}$$

$$\sin\phi_p = \frac{\tan\delta}{\tan\theta}, \tag{10}$$

$$b^2 = -\frac{\partial\varepsilon}{\Delta\varepsilon}. \tag{11}$$

Most commercial S_C materials have $\Delta\varepsilon<0$ and $\partial\varepsilon>0$ [12], therefore we consider only the case $b^2>0$.

Using equation (4) and defining

$$\alpha = \frac{\varepsilon_0\Delta\varepsilon V}{P_s d}, \tag{12}$$

the response time is given by

$$\left(\frac{P_s^2\cos\delta}{\varepsilon_0\Delta\varepsilon\eta}\right)\tau = \frac{1}{\alpha}\int_{\phi_0}^{\pi/2} \frac{d\phi}{\sin\phi+\alpha[(\sin^2\theta+b^2)\cos\delta\sin\phi-\sin\theta\cos\theta\sin\delta]\cos\phi}. \tag{13}$$

We integrate this equation numerically and look for a minimum in α to investigate the effect of $\Delta\varepsilon$, $\partial\varepsilon$ and δ on $V_{\min}$, the voltage which minimizes the switching time between ϕ_0 and $\phi=\pi/2$.

For the system when $\delta=\theta$, figure 1 shows the effect of the dielectric anisotropies on $V_{\min}$ when switching from the alternating field stabilized position. Clearly a lower $V_{\min}$ is achieved with a large positive $\partial\varepsilon$ and a small negative $\Delta\varepsilon$, further $V_{\min}$ is insensitive to $\Delta\varepsilon$ if $\partial\varepsilon$ is large ($\geqslant 0{\cdot}4$). In the bookshelf geometry both the negative $\Delta\varepsilon$ and positive $\partial\varepsilon$ stabilize the fully switched position; but if $\delta=\theta$, $\Delta\varepsilon$ acts to stabilize the $\phi=\pi/2$ position.

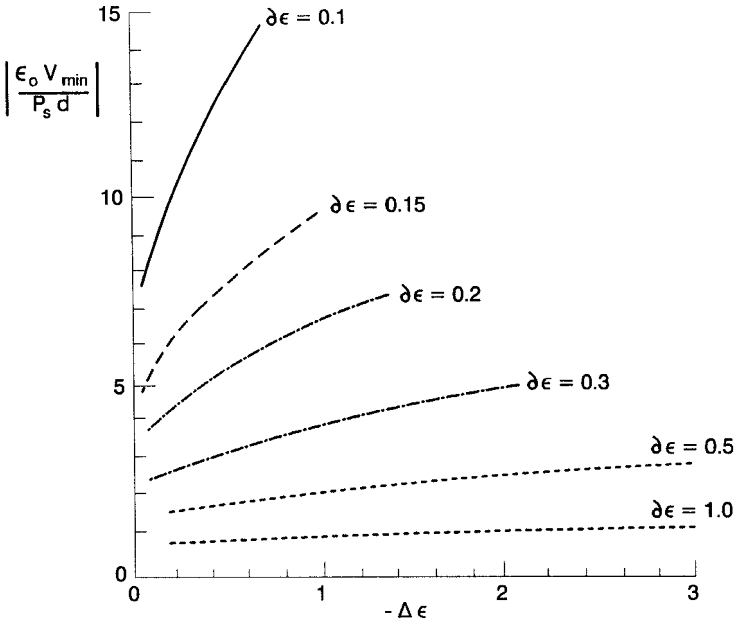

Figure 1. V_{min} as a function of $\Delta\varepsilon$ for various $\partial\varepsilon$, with $\delta=\theta=22°$.

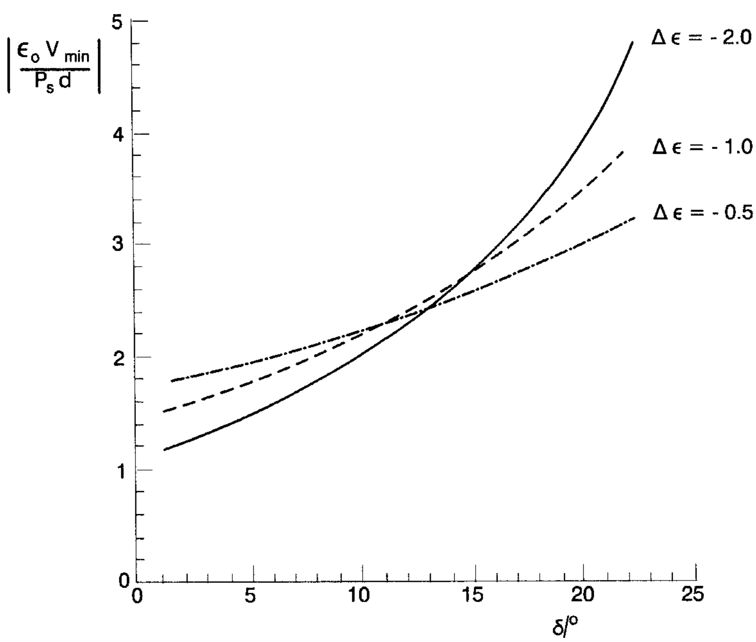

Figure 2. V_{min} as a function of δ for various $\Delta\varepsilon$ with $\theta=22°$, $\partial\varepsilon=0{\cdot}3$.

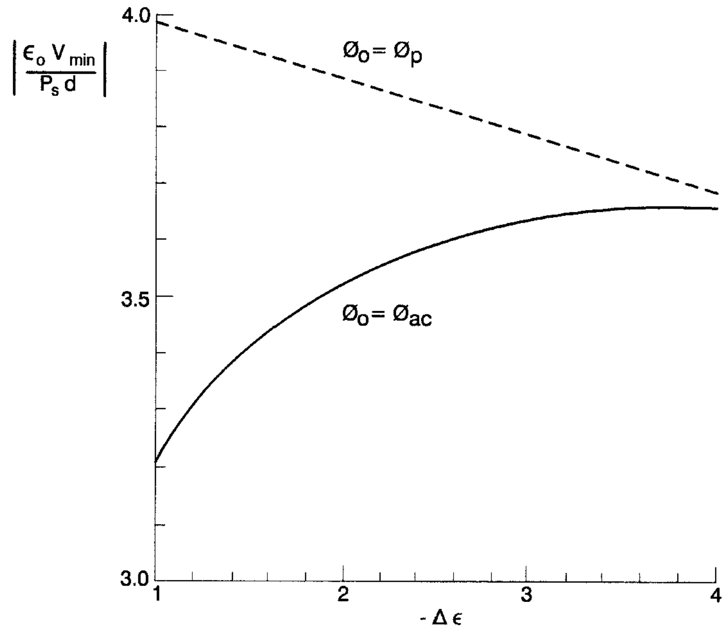

Figure 3. V_{min} as a function of $\Delta\varepsilon$ with $\delta = 18°$, $\theta = 22°$ and $\partial\varepsilon = 0{\cdot}3$ switching from either the AC field stabilized position or the in-plane position.

In general then the effect of $\Delta\varepsilon$ on V_{min} depends upon the layer tilt angle and this is illustrated in figure 2. In practice a surface stabilized ferroelectric liquid crystal does not switch from ϕ_{ac}, a comparison to switching from ϕ_p is shown in figure 3 for typical values of δ and θ. Essentially experimental values of V_{min} will decrease from the ϕ_p value to the ϕ_{ac} value as the applied alternating field (the data voltage) is increased.

Although we have had to calculate V_{min} numerically for the general case, a useful approximation can be deduced from the voltage (V_{div}) at which the response time diverges. When switching from $\phi = \phi_{ac}$ to $\phi = \pi/2$ V_{div} is the lowest voltage at which the torque becomes zero for any $\phi \in (\phi_{ac}, \pi/2)$, from equation (4)

$$\left| V_{div} \atop \phi_{ac} \to \frac{\pi}{2} \right| = \left| \frac{P_s d}{\varepsilon_0 \cos\delta((\Delta\varepsilon \sin^2\theta - \partial\varepsilon)^{2/3} - (\Delta\varepsilon \sin\theta \cos\theta \tan\delta)^{2/3})^{3/2}} \right|. \tag{14}$$

For a range of typical values of $\Delta\varepsilon$, $\partial\varepsilon$, δ and θ, comparison of the numerical results for V_{min} with V_{div} show $0{\cdot}6 < V_{min}/V_{div} < 0{\cdot}65$. Thus, we suggest that in many cases V_{min} can be usefully approximated by

$$\left| V_{min} \atop \phi_{ac} \to \frac{\pi}{2} \right| \approx 0{\cdot}62 \left| V_{div} \atop \phi_{ac} \to \frac{\pi}{2} \right|, \tag{15}$$

to within 5 per cent accuracy.

3. Discussion

Materials showing the S_C–S_A–N phase sequence generally have the chevron layer structure in parallel aligned cells, but it is clear from the theory of Xue *et al.* [15] that, in tilted layer geometries, uniaxial ferroelectric liquid crystals with typical negative values of $\Delta\varepsilon$ will not show measurable minima in the response time–voltage characteristic. We have shown that the τ–V minimum occurs in tilted systems due to biaxiality in the dielectric permittivity tensor, and in view of this we suggest that the results of Saunders *et al.* [13] may be reinterpreted.

Restricting ourselves to the case of $\Delta\varepsilon < 0$ and $\partial\varepsilon > 0$ we have shown in the general case that, for a given $\Delta\varepsilon$, V_{min} can be reduced by increasing $\partial\varepsilon$; the development of materials with large $\partial\varepsilon$ is therefore recommended for use with multiplexing schemes which utilize the minimum in the τ–V curve. In a biaxial liquid crystal the three principal permittivity components depend upon four order parameters and the square of each of the dipole components [16]. It should, therefore, be possible to vary $\partial\varepsilon$ and $\Delta\varepsilon$ by using suitable combinations of polar groups, although these may affect the phase stability.

In this paper we have restricted ourselves to $\Delta\varepsilon < 0$, however the case of $\Delta\varepsilon > 0$ is also worth investigating, with $\Delta\varepsilon \sin^2\theta = \partial\varepsilon$ being a simple example of this. A more complete non-uniform switching model should also be developed based upon the dynamic theory of S_C liquid crystals [17]. A comparison of the current calculations to experiment is in progress.

The authors would like to thank J. R. Hughes and H. J. Hutchinson for useful discussions.

Appendix

Consider the coordinate system in figure A. The liquid crystal configuration is described by the following orthonormal vectors:

the layer normal $\mathbf{a} = (\cos\delta, 0, \sin\delta)$;

a vector along the C_2 axis $\mathbf{b} = (-\sin\delta\cos\phi, \sin\phi, \cos\delta\cos\phi)$;

the c director $\mathbf{c} = (\sin\delta\sin\phi, \cos\phi, -\cos\delta\sin\phi)$.

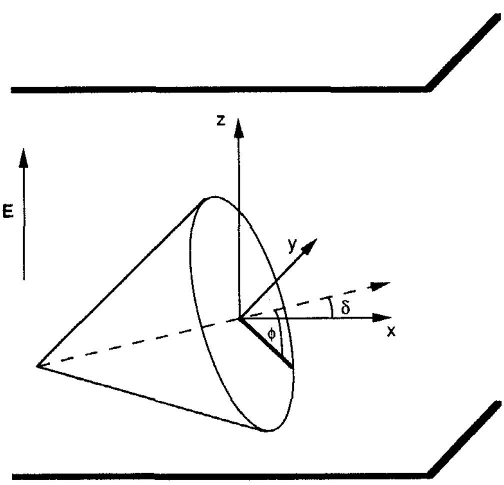

Figure A. The smectic C coordinate system.

Then letting

$$\mathbf{n}=\mathbf{a}\cos\theta+\mathbf{c}\sin\theta$$

and

$$\mathbf{k}=\mathbf{n}\wedge\mathbf{b}=\mathbf{a}\sin\theta-\mathbf{c}\cos\theta$$

the dielectric permittivity tensor

$$\varepsilon_{ij}=\varepsilon_1 k_i k_j+\varepsilon_2 b_i b_j+\varepsilon_3 n_i n_j.$$

Assuming the layers are fixed we need only consider the torque acting to rotate the $\mathbf{n}$ director around the cone, then if electric torques dominate over elastic torques the reorientation equation is thus

$$-\eta\frac{d\phi}{dt}=P_s(\mathbf{b}\wedge\mathbf{E})\cdot\mathbf{a}+\varepsilon_0((\varepsilon\mathbf{E})\wedge\mathbf{E})\cdot\mathbf{a}=P_sEa_xb_y+\varepsilon_0E^2a_x\varepsilon yz,$$

where $\mathbf{E}=E(0,0,1)$ giving

$$\eta\frac{d\phi}{dt}=P_s\frac{V}{d}\cos\delta\sin\phi+\varepsilon_0\frac{V^2}{d^2}$$
$$\times\left[(\Delta\varepsilon\sin^2\theta-\partial\varepsilon)\cos^2\delta\sin\phi\cos\phi-\frac{\Delta\varepsilon}{4}\sin 2\theta\sin 2\delta\cos\phi\right].$$

References

[1] Meyer, R. B., 1977, *Molec. Crystals liq. Crystals*, **40**, 33.
[2] Clark, N. A., and Lagerwall, S. T., 1980, *Appl. Phys. Lett.*, **36**, 899.
[3] Rieker, T. P., Clark, N. A., Smith, G. S., Parmar, D. S., Sirota, E. B., and Safinya, C. R., 1987, *Phys. Rev. Lett.*, **59**, 2658.
[4] Anderson, M. H., Jones, J. C., Raynes, E. P., and Towler, M. J., 1991, *J. Phys.* D, **24**, 338.
[5] Dijon, J., Ebel, C., Vauchier, C., Baume, F., Clerc, J., Estor, M., Leroux, T., Maltese, P., and Mulatier, L., 1988, *SID Digest 246*.
[6] Yang, K. H., and Chieu, T. C., 1989, *Jap. J. appl. Phys.*, **28**, L1599.
[7] Maclennan, J. E., Clark, N. A., Handschy, M. A., and Meadows, M. R., 1990, *Liq. Crystals*, **7**, 753.
[8] Orihara, H., Nakamura, K., Ishibashi, Y., Yamada, Y., Yamamoto, N., and Yamawaki, M., 1986, *Jap. J. appl. Phys.*, **25**, 839.
[9] Bone, M. F., Coulson, I., Hughes, J. R., Ross, P. W., Saunders, F. C., and Surguy, P. W. H., European Patent Application No. 88 307 804.0.
[10] Ross, P. W., 1988, *720 × 400 Matrix Ferroelectric Display Operating at Video Frame Rate* (Proc. IDRC), p. 185.
[11] de Gennes, P. G., 1974, *The Physics of Liquid Crystals* (Oxford University Press), p. 307.
[12] Jones, J. C., Raynes, E. P., Towler, M. J., and Sambles, J. R., 1991, *Molec. Crystals liq. Crystals*, **199**, 277.
[13] Saunders, F. C., Hughes, J. R., Pedlingham, H. A., and Towler, M. J., 1989, *Liq. Crystals*, **6**, 341.
[14] Xue Jiu-Zhi., Handschy, M. A., and Clark, N. A., 1987, *Ferroelectrics*, **73**, 305.
[15] Xue Jiu-Zhi., Handschy, M. A., and Clark, N. A., 1987, *Liq. Crystals*, **2**, 707.
[16] Toriyama, K., Dunmur, D. A., and Hunt, S. E., 1989, *Liq. Crystals*, **5**, 1001.
[17] Leslie, F. M., Stewart, I. W., and Nakagawa, M., 1991, *Molec. Crystals liq. Crystals*, **198**, 443.